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CKM angles from non-leptonic B decays using SU(3) flavour symmetry

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Abstract. We discuss the determination of the CKM angles γ and α using recent data from non-leptonic B decays together with flavour symmetries. Penguin effects are controlled by means of the CP-averaged branching ratio $B_d \to \pi^\pm K^\mp$. The information from $\mathcal{A}_{CP}(B_d \to J/\Psi K_S)$ (two solutions for ϕ_d), R_b and γ allow us to determine β, even in presence of New Physics not affecting $\Delta B = 1$ amplitudes. In this context we address the question of to what extent there is still space for New Physics.

PACS. 13.25Hw Hadronic decays of mesons – 11.30Er CP violation

1 Introduction

B physics is one of the most fertile testing grounds to check the CKM mechanism of CP violation in the SM [1], but also to look for the first signals of New Physics [2] in the pre-LHC era.

The huge effort at the experimental level at the B factories and future hadronic machines [3] has produced, already, several impressive results. First, the measurement of $\sin\phi_d$ from the mixing induced CP asymmetry of the decay $B_d \to J/\Psi K_S$. Second, the measurement of a series of non-leptonic B decays: $B_d \to \pi K$, $B_d \to \pi\pi$ and in the future hadronic machines $B_s \to KK$ will be also accessible.

These non-leptonic B decays play a fundamental role in the determination of the CKM angle γ. The main problem in analyzing them is how to deal with hadronic matrix elements and how to control penguin contributions. Our approach [4,5,6,7] extract the maximal possible information from data using flavour symmetries to try to reduce as much as possible the uncertainties associated to QCD hypothesis.

2 CKM angle γ from non-leptonic decays: $B_d \to \pi\pi$, $B_d \to \pi K$ and $B_s \to KK$

We start writing down a general amplitude parametrization of $B_d \to \pi^+\pi^-$ in the SM [4,6]:

$$A(B_d^0 \to \pi^+\pi^-) = \mathcal{C}\left(e^{i\gamma} - \mathbf{d}e^{\mathbf{i}\theta}\right)$$

All the hadronic information is collected in

$$de^{i\theta} \equiv \frac{1}{R_b}\left(\frac{A^{ct}_{\rm pen}}{A^u_{\rm CC} + A^{ut}_{\rm pen}}\right) \quad \mathcal{C} \equiv \lambda^3 A\, R_b\left(A^u_{\rm CC} + A^{ut}_{\rm pen}\right)$$

where $A^u_{\rm CC}$ are current-current contributions and $A^{qt}_{\rm pen}$ are differences between penguin contributions with a quark $q = u, c$ and a quark top inside the loop.

This amplitude allow us to construct the corresponding CP asymmetries [4,6]:

$$\mathcal{A}^{\rm dir}_{\rm CP} = {\rm func}(d,\theta,\gamma) \quad \mathcal{A}^{\rm mix}_{\rm CP} = {\rm func}(d,\theta,\gamma,\phi_d)$$

Following a similar procedure we can write down the amplitude for a closely related process:

$$A(B_s^0 \to K^+K^-) = \left(\frac{\lambda}{1-\lambda^2/2}\right)\mathcal{C}'\left[e^{i\gamma} + \left(\frac{1-\lambda^2}{\lambda^2}\right)d'e^{i\theta'}\right]$$

whose corresponding asymmetries will depend on [4,6]:

$$\mathcal{A}^{\rm dir}_{\rm CP} = {\rm func}(d',\theta',\gamma) \quad \mathcal{A}^{\rm mix}_{\rm CP} = {\rm func}(d',\theta',\gamma,\phi_s)$$

The crucial point, here, is that the hadronic parameters d', θ' and $\mathcal{C}'$, has exactly the same functional dependence on the penguins that d, θ and $\mathcal{C}$, except for the interchange of a d quark by an s quark.

As a consequence, both processes can be related via U-spin symmetry, reducing the total number of parameters to five: γ, d, θ, ϕ_d and ϕ_s. At this point, one must check the sensitivity of the results to the breaking of U-spin symmetry. This is explained in Sect. 2.2.

Looking a bit more in detail, one finds that d is indeed not a free parameter, but it can be constrained or substituted using an observable called H [7,6]:

$$H \equiv \frac{1}{\epsilon}\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|^2\left[\frac{M_{B_d}}{M_{B_s}}\frac{\Phi(\frac{M_K}{M_{B_s}},\frac{M_K}{M_{B_s}})}{\Phi(\frac{M_\pi}{M_{B_d}},\frac{M_\pi}{M_{B_d}})}\frac{\tau_{B_s}}{\tau_{B_d}}\right]\left[\frac{{\rm BR}(B_d \to \pi^+\pi^-)}{{\rm BR}(B_s \to K^+K^-)}\right]$$

This quantity requires the knowledge of ${\rm BR}(B_s \to K^+K^-)$, which is still not available. However, we can already now evaluate H by making contact with the B factories and substitute $B_s \to K^+K^-$ by $B_d \to \pi^\pm K^\mp$. These two processes differ by the spectator quark and certain exchange and penguin annihilation topologies that are expected to be small [8]. This leads to the following value for H [9]:

$$H \approx \frac{1}{\epsilon}\left(\frac{f_K}{f_\pi}\right)^2 \left[\frac{\mathrm{BR}(B_d \to \pi^+\pi^-)}{\mathrm{BR}(B_d \to \pi^\mp K^\pm)}\right] = 7.5 \pm 0.9 \qquad (1)$$

Due to the dependence of H only on $\cos\theta\cos\gamma$ in the U-spin limit, we obtain immediately a constrained range for d: $0.2 \le d \le 1$. Also, using the exact expression for H we can obtain d as a function of H, θ and γ.

It is important to insist here that once the data on the branching ratio of $B_s \to KK$ will be available, the spectator quark hypothesis will not be necessary and only U-spin breaking effects will be important.

2.1 Prediction for CKM-angle γ

Let's take as starting point the general expression [6]:

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+\pi^-) = \mp\left[\frac{\sqrt{4d^2 - (u+vd^2)^2}\,\sin\gamma}{(1-u\cos\gamma)+(1-v\cos\gamma)d^2}\right] \qquad (2)$$

where $u, v, d = F_i(\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}, H, \gamma, \phi_d(B_d \to J/\Psi K_s); \xi, \Delta\theta)$. The parameters ξ, $\Delta\theta$ will account for the U-spin breaking and are discussed in Sect. 2.2.

Using present world average for $\sin\phi_d = 0.734 \pm 0.054$, one obtains two possible solutions for the weak mixing angle:

$$\phi_d = \left(47^{+5}_{-4}\right)^\circ \vee \left(133^{+4}_{-5}\right)^\circ .$$

We will refer later on to these two solutions like scenario A and B, respectively.

Concerning experimental data, the situation is still uncertain, but improving. Present naive average of Belle and Babar data is [10]:

$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}(B_d \to \pi^+\pi^-) = -0.38 \pm 0.16$$
$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to \pi^+\pi^-) = +0.58 \pm 0.20$$

The intersection of the two experimental ranges of $\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}$ and $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}$ allow us, using (2), to determine the range for γ. The first range, corresponding to take $\phi_d = 47^\circ$ is:

$$32^\circ \lesssim \gamma \lesssim 75^\circ \qquad (3)$$

For the second solution $\phi_d = 133^\circ$ one obtains:

$$105^\circ \lesssim \gamma \lesssim 148^\circ \qquad (4)$$

Both plots are symmetric (see [6,11]). This is a consequence of the symmetry $\phi_d \to 180^\circ - \phi_d$, $\gamma \to 180^\circ - \gamma$ that (2) exhibits. It is remarkable the stability of the range for γ if we compared it with previous analysis [11].

2.2 Sensitivity to parameters H, ξ, and $\Delta\theta$

Here we will analyze the sensitivity of the determination of γ on the variation of the different hadronic parameters.

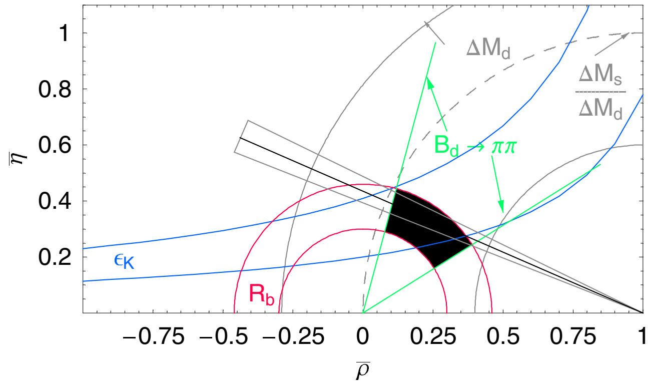

Fig. 1. $\phi_d = 47^\circ$. SCENARIO A

2.2.1 H and the spectator quark hypothesis

Let's fix the solution $\phi_d = 47^\circ$ and take the experimental branching ratios of $B_d \to \pi\pi$ and $B_d \to \pi K$ to determine H. We vary H inside its experimental range (1) at one, two and three sigmas to take into account the uncertainty associated to the spectator quark hypothesis. We find at one sigma a very mild influence in the determination of γ. The error induced in the range of γ is about $\pm 2^\circ$.

For the very conservative range of up to three sigmas we find a maximal error of 6°. Moreover, if the experimental value of H tends to increase the range for γ tends to decrease, allowing for a narrower determination.

Finally, the uncertainty associated to H will be drastically reduced once the $\mathrm{BR}(B_s \to KK)$ is known and H will be taken safely in a narrower range.

2.2.2 U-spin breaking: ξ and $\Delta\theta$

U-spin breaking is the most important uncertainty. We will follow two different strategies to keep it under control:

a) Once the data from the CP asymmetries and branching ratio of $B_s \to KK$ will be available and ϕ_s will be measured from the CP-asymmetry of $B_s \to J/\Psi\phi$, we will be able to *test* directly from data U-spin breaking. Taking ϕ_d from $B_d \to J/\Psi K_S$ we will have 4 observables (the CP asymmetries) and 3 unknowns (d, θ, γ). Then, we can add d' as another free parameter and data will tell us the amount of U-spin breaking.

b) Already now, we can define two quantities $\xi = d'/d$ and $\Delta\theta = \theta' - \theta$ that parametrizes the amount of U-spin breaking. In order to test the sensitivity of γ to the variation of these parameters, we allow them to vary in a range. If we allow for a very large variation of ξ between 0.8 and 1.2, the larger error in the determination of γ is of $\pm 5^\circ$. Concerning $\Delta\theta$, its influence is negligibly small, a variation of 40° induces an error of at most 1 degree.

Other studies on U-spin breaking can be found in [12].

3 Determination of CKM angles α and β in SM and with new physics in the mixing

Next point is how to determine α and β [9]. Here, in addition, we will also allow for Generic New Physics affecting the B_d^0–$\overline{B_d^0}$ mixing, but not to the $\Delta(B,S)=1$ decay amplitudes, i.e, this type of New Physics is consistent with the determination of γ explained in the previous section. Our inputs are[9,13]:

- $R_b \equiv \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right|$ obtained from exclusive/inclusive transitions mediated by $b \to u\ell\overline{\nu}_\ell$ and $b \to c\ell\overline{\nu}_\ell$. Two important remarks are: a) This is an observable practically insensitive to New Physics, b) from $R_b^{\max}=0.46$ we can extract a robust maximum possible value for β: $|\beta|_{\max}=27°$, respected by the two scenarios.
- γ obtained as discussed in previous sections.
- ϕ_d from $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}(B_d \to J/\psi K_{\mathrm{S}})$ is used as an input for the CP asymmetries of $B_d \to \pi\pi$, but NOT to determine β, since we assume that New Physics could be present. Also ΔM_d and $\Delta M_s/\Delta M_d$ are not used as inputs, due to their sensitivity to New Physics.

Using these inputs we obtain two possible determinations for α, β and γ, corresponding to the two possible values of ϕ_d.

3.1 Scenario A: Compatible with SM

This scenario corresponds to the first solution $\phi_d=47°$, which implies the range for γ given in (3). Together with R_b we obtain the black region shown in Fig. 1. It implies the following prediction for the CKM angles:

$$78° \leq \alpha \leq 136° \quad 13° \leq \beta \leq 27° \quad 32° \leq \gamma \leq 75°$$

and the error associated with $\xi \in [0.8, 1.2]$ is $\Delta\alpha = \pm 4°$, $\Delta\beta = \pm 1°$ and $\Delta\gamma = \pm 5°$. It is interesting to notice that this region is in good agreement with the usual CKM fits [14]. To illustrate it we have shown in Fig. 1 also the prediction from the SM interpretation of different observables: ΔM_d, $\Delta M_s/\Delta M_d$, ϵ_K and $\phi_d^{SM}=2\beta$.

3.2 Scenario B: New Physics

The second solution: $\phi_d = 133°$ *cannot* be explained in the SM context and requires New Physics contributing to the mixing[9,13]. Models with New sources of Flavour mixing can account for this second solution with only two very general requirements [9]: a) The effective scale of New Physics is larger than the electroweak scale and b) the adimensional effective coupling ruling $\Delta B=2$ processes can always be expressed as the square of two $\Delta B=1$ effective couplings. Supersymmetry provides a perfect example, in particular, through the contribution of gluino mediated box diagrams with a mass insertion $\delta^D_{bL\,dL}$[9].

In this case, γ lies in the second quadrant (4) and β is indeed smaller than in the previous scenario. The result

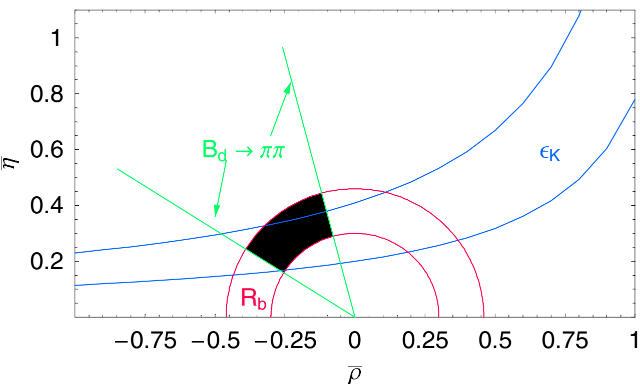

Fig. 2. $\phi_d = 133°$. SCENARIO B

is still consistent with the ϵ_K hyperbola. $\Delta M_{d,s}$ are not shown here, since they would be affected by New Physics. The black region obtained (see Fig. 2) corresponds to the following prediction for the CKM angles:

$$22° \leq \alpha \leq 60° \quad 8° \leq \beta \leq 22° \quad 105° \leq \gamma \leq 148°$$

with same errors associated to ξ as in Scenario A. It is interesting to remark that this second solution has also interesting implications for certain rare decays like $K^+ \to \pi^+\nu\overline{\nu}$[9,15]. Using this second solution we find a better agreement with experiment than with the SM solution. Concerning $B_d \to \mu^+\mu^-$, we find also sizeable differences depending on the scenario used.

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